① kth order statistics

[23, 14, 54, 67, 12] ① , [3, 2, 1, 68, 89] ② , [31, 13, 34, 45] ③

[7, 9, 22, 11, 29] ④ , [38, 78, 19, 17, 71] ⑤ ,

[18, 81, 91, 92, 26] ⑥ [100, 44] ⑦

$\lfloor \frac{n}{5} \rfloor$ groups $\Rightarrow \lfloor \frac{32}{5} \rfloor = 6$ (5 element in each group)

group 7 $\rightarrow$ $n \bmod 5$

$32 \bmod 5 = 2$, element

② Find median of each groups: apply insertion sort & Find median; it takes O(1) time

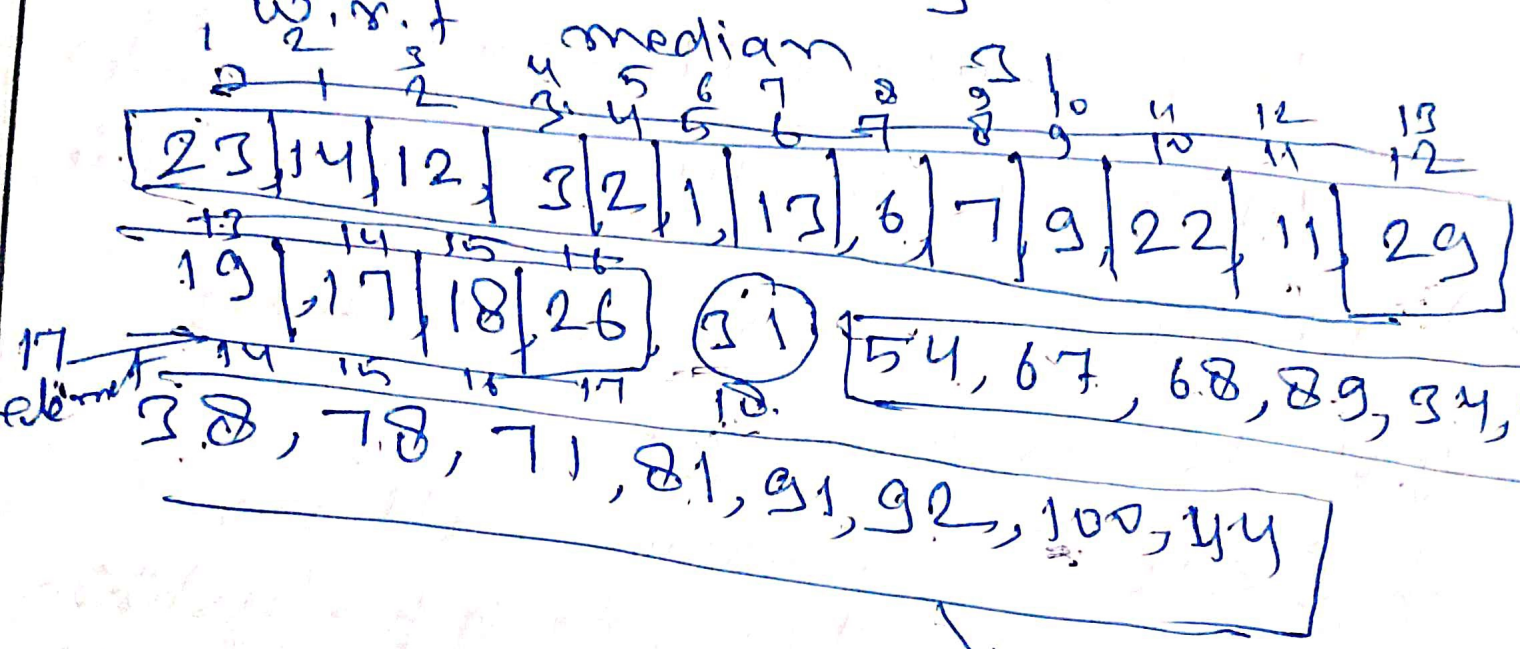
12	1	6	7	17	18	44
14	2	13	9	19	26	100
(23)	(3)	(31)	(11)	(38)	(81)	
54	68	34	22	71	91	
67	89	45	29	78	92	

(3) make the array of median

23 3 (31) 11 79 81

median of medians is 31

(4) Partition in original Dataset w.r.t median



14 elements

$\left(\frac{3n}{10} - 6\right)$ elements on either side

$\Rightarrow \frac{3 \times 12}{10} - 6 \Rightarrow 3.6$ element

(5) Using this modified version of partition that takes pivot element as input;

Let x be the k^{th} element of the array after partitioning then lowside have k-1 elem
highside have n-k elem

In example

if x = 31 k = 18

k-1 ⇒ 18-1 = 17 element (low side)

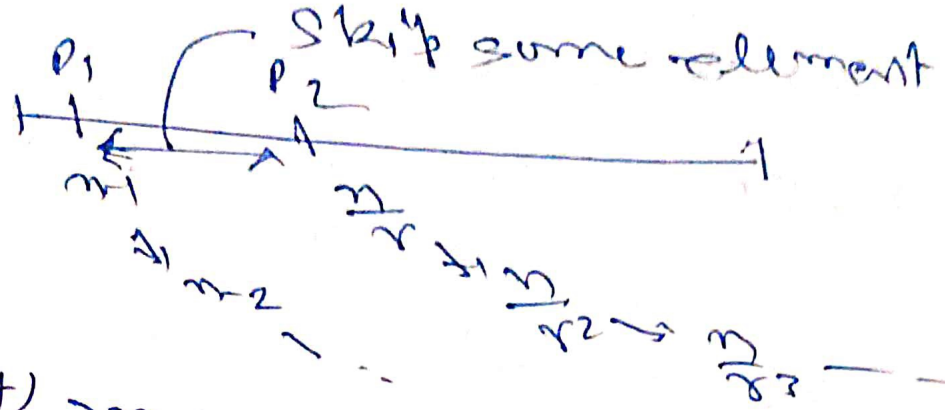
n-k ⇒ 32-18 = 14 in high side

(6) There are three possibilities

if i = k return x

if i < k recursive call in lowside of for partition

if i > k return (return i^{th} smallest) $(i-k)^{th}$ smallest in high side and recursive call of partition



$D(\text{element}) \Rightarrow n + n-1 + n-2 + \cdots = O(n^2)$

$\Rightarrow n + \frac{n}{r} + \frac{n}{r^2} + \cdots \Rightarrow \left(\frac{1}{1-r}\right)n$

$\Rightarrow C(n)$

$\Rightarrow O(n)$

Hence median of median concept.

7	10	4	3	20	15

$k = 3$ $\quad O/P \Rightarrow 7$

3rd smallest element.

1	2	3	4	5	6
3	4	7	10	15	20

$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + O(n)$$

$\Rightarrow O(n)$